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Inverse Functions

An inverse function is the result of interchanging the dependant and independent variables in an equation of a function. For example, the inverse function of *y = 2x* is *x = 2y*, or *y = x/2*. The inverse function of *f* only exists if f is one-to-one. That is, not only must each x have only one corresponding *y*, but no two values of x may have the same functional value. This is rather obvious since the interchange of *x* and *y* means that if *f* has two values of x with the same functional value, *f-1*will have two values of y for the same x, making it simply a relation and not a function.

The function *y = x* is rather a special one because its inverse is equal to itself. It’s also unique because, like the lines *y = 0* and *x = 0*, it is a reflecting line. We can reflect a function about the y-axis by substituting x for *–x*. What that essentially means is that we’re replacing x with its mirror image. The same can be said of reflecting about the x-axis. A similar thing happens when x is replaced with y. Since the line *y = x* is the “intersection” of sorts where x and y meet, *f-1* is the mirror image of *f* about *y = x*. Just like 0 is the “intersection” of –x and x – where the two meet – *y = x* is the intersection and thus the line about which *f-1* is reflected.

Theorem 3.17 says that the slope of *g(x),* where *g(x) = f-1(x),* is the reciprocal of the slope of *f(g(x))*. That is, if slope of *f* at (1,5) is 5, the slope of *g* at (5,1) is 1/5. That’s a really useful fact. Technically speaking, it means that *g’(x) = 1/(f’(g(x))*. For example, say we want to find the slope of *g* at *x=3*. We know that it’s going to be the reciprocal of the slope of *f* where *y=3* and *x=g(3)*. Ergo, the slope of *g* at *x=3* is the reciprocal of *f’(g(x)*.